

GRANULOMETRIC DETERMINATION OF SEDIMENTARY ROCK PARTICLE ROUNDNESS

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Abstract.

A number of subjective and objective methods have been proposed, and are in use, to determine the roundness of sedimentary rock particles. Roundness, which is one of three properties describing the shape of a particle, is a measure of the extent to which the corners and edges of a particle have been worn away.

The objective roundness measures are mainly based on the Fourier transform. In this paper granulometric methods are proposed as alternative objective methods. In fact, we show that even a single morphological opening with a disk of adequately chosen radius can provide a roundness measure that is highly correlated with the “objective” roundness, as provided by Krumbein’s chart [5]. In addition, we propose an efficient alternative method based on a linear opening of the polar representation of the boundary of the particles considered. In our experiments, both methods compared favorably with the Fourier transform method of Diepenbroek *et al.* [3].

Key words: Fourier Transform, Granulometries, Krumbein chart, Morphology, Opening, Roundness, Rock Particles.

1. Introduction

In order to construct a mathematical model of an observed phenomenon, one first has to be able to measure that which is observed. This paper proposes a method of measuring one of the shape properties of sedimentary particles in order that the transportation history of those particles as well as of their behavior in hydrodynamic systems can be modelled [5].

Three independent properties are typically used to describe the shape of a particle [1]:

Form describes the overall shape or sphericity of a particle; it is a measure of the extent to which a particle approaches a sphere in shape.

Roundness is a measure of the extent to which the edges and corners of a particle has been rounded.

Surface texture describes the markings on the surface of a particle.

A number of subjective as well as objective methods have been proposed to determine the roundness of rock particles. Two alternative objective methods, both based on morphological openings, are proposed in this paper.

Wadell [17] proposed that roundness be quantitatively defined as the ratio of the mean radius of curvature of the corners to the radius of the maximum inscribed circle:

$$\frac{\frac{1}{N} \sum_{i=1}^N r_i}{R}$$

where r_i is the radius of curvature of the i -th corner and R is the radius of the maximum inscribed circle. A number of other quantitative measures have been proposed. However, it is only with the procedure proposed by Wadell [17] that mean roundness can be measured.

A number of pebble comparison charts have also been developed to aid in the determination of roundness. Examples of these charts are those proposed by Krumbein [6], Powers [10], and Russell and Taylor [11]. Of the charts available for visual estimation, Krumbein's is preferred as it has the largest number of roundness classes [1]. A subset of the Krumbein chart is shown in Fig. 1. The argument for using the chart with the largest number of classes is that it is closest to a continuous measure. It is however required that adjacent classes be distinguishable [1]. The chart of Krumbein [6] consists of 81 profiles divided into 9 roundness classes from 0.1 to 0.9. Each class therefore contains 9 profiles. (The profiles in Fig. 2 are representative of the rest of the chart.)

1.1. FOURIER BASED METHODS

A number of authors have proposed methods using the Fourier transform to determine roundness [3, 4, 12]. The basic principle is to obtain coordinates on the edge of the profile of the fragment being measured (See Fig. 2). A center point is determined, and all the edge coordinates converted to polar coordinates using the center point as origin. For example, the polar coordinates of the fragments of Fig. 2 are shown in Fig. 3. The Fourier transform of the vector of radii (ρ), or ρ -curve, is then calculated and the roundness determined from the Fourier transform coefficients.

1.2. THE DIEPENBROEK METHOD

Diepenbroek *et al* [3] make use of the sum of the amplitudes of the first 24 coefficients of the Fourier transform. To compensate for different size rock fragments, the coefficients are divided by the zero-th coefficient, that is, by the mean radius of the rock fragment. The ellipse which best approximates the outline of the rock fragment is obtained from the second Fourier coefficient.

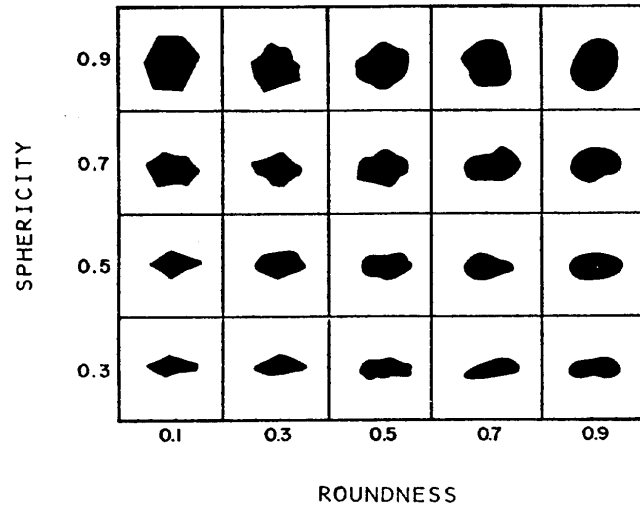


Fig. 1. Subset of Krumbein particle roundness/sphericity chart.



Fig. 2. Profile of rock fragments with roundness values of (A) 0.1 and (B) 0.9.

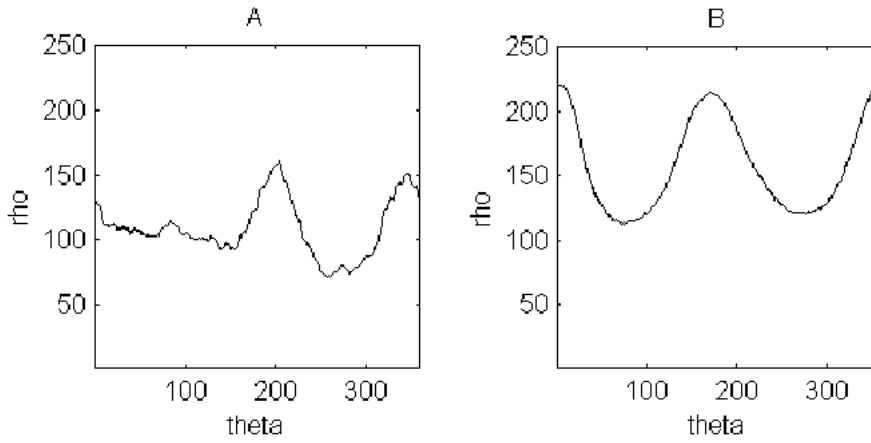


Fig. 3. Polar coordinates of the rock fragments in Fig. 2.

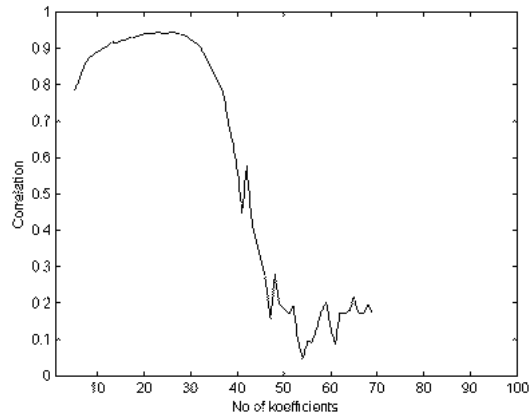


Fig. 4. Correlation between the Diepenbroek and actual roundness values.

By subtracting this ellipse's Fourier transform from that of the rock fragment the method is made insensitive to sphericity. A further refinement, proposed by Diepenbroek *et al* in [3], is an empirical method for removing the effects of digitization or pixel noise.

The method of Diepenbroek *et al* was implemented and its results were used in comparison against the results obtained with the proposed methods. The 81 rock profiles of the comparison chart of Krumbein [6] provided the test data we used in our experiments: after digitization and binarization, edge pixel coordinates were turned into polar coordinates using the centroid (center of mass) of each profile as origin. Interpolation was used to normalize the data and obtain 1,024 polar coordinates with evenly spaced θ values for each profile, as is shown in Fig. 3.

The method of Diepenbroek *et al* was then applied using the first N coefficients of the Fourier transform for $N = 5, \dots, 100$. The correlation between the Diepenbroek and the actual roundness values (as obtained from the digitized chart) is shown in Fig. 4. This experiment gave empirical evidence that using $N = 5, \dots, 24$ provided the best correlation between the Diepenbroek computed roundness and the actual roundness. The relationship between the Diepenbroek roundness computed for $N = 5, \dots, 24$ and the actual roundness values is shown in Fig. 5. A correlation of 0.94 was obtained with this method.

2. Granulometric Roundness Analysis

The concept of a granulometry was introduced by George Matheron in the late sixties as a new tool for studying porous media [8, 9]. The size of the pores in such media was characterized using series of openings with structuring elements of increasing size [13]. Since these early days, the properties of granulometries have been studied in numerous papers. Granulometries have also been used in a wide variety of image analysis tasks that include feature extraction, segmen-

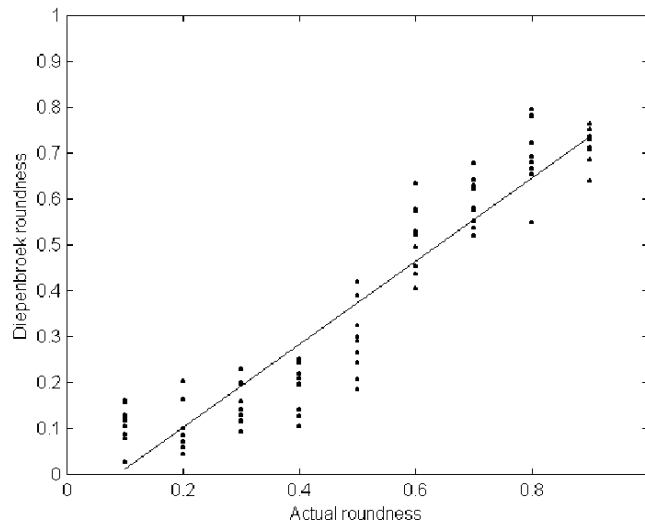


Fig. 5. Relationship between Diepenbroek-computed roundness and actual roundness, with correlation of 0.94.

tation, texture description and size estimation (See [16] for some examples of application).

Given a granulometric family of openings $\Gamma = (\gamma_n)_{n=1,N}$, performing the granulometric analysis of a binary image I with Γ typically consists of measuring the area (i.e., number of pixels) of $\gamma_n(I)$, for $n = 1, \dots, N$. The function mapping every openings size n to the area of $\gamma_n(I)$ can then be differentiated and normalized, resulting in a curve that is often called *granulometric curve* or *pattern spectrum* of I . The term “pattern spectrum” was coined by Maragos in a 1989 a paper [7], and highlights the connection between Fourier analysis and granulometries.

2.1. USE OF CIRCULAR OPENINGS

In this paper, given that our objective is to study particle roundness, we considered granulometries with (Euclidean) disks of increasing size. For the sake of full disclosure, we should point out that the family of openings with discrete approximation of Euclidean disks of increasing radius is not actually a granulometry: indeed, if D_n represents the discrete approximation of a disk of radius n , there are many values p, q for which $D_{p+q} \neq D_p \oplus D_q$. However, this approximation was sufficient for our purpose.

As input, we used the 81 digitized particle images of the Krumbein chart [6]. Circular structuring elements with radii varying between 1 and that of the largest inscribed circle of each particle were used. To speed up computation, we did not go through a series of openings with disks of increasing size, but instead, implemented a Euclidean extension to the openings function algorithms proposed in [15]. The opening function for the particle Fig. 2a is shown in Fig. 6.



Fig. 6. Euclidean opening function for particle shown in Fig. 2a.

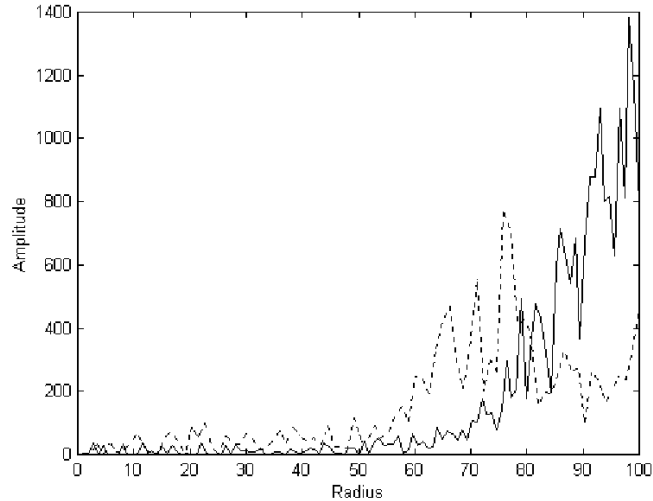


Fig. 7. Granulometric curves (pattern spectra) of particle a (dotted line) and particle b (solid line) in Fig. 2. Radius is given as a percentage of the radius of the largest inscribed circle of each particle.

The unnormalized granulometric curve can be obtained by simple histogramming of the opening function. The final pattern spectrum is then obtained by:

1. Normalizing the radii to take values between 1 and 100.
2. Normalizing granulometric values through division by the total area of the particle.

The pattern spectra of the particles of Fig. 2 are shown in Fig. 7. In these curves, the amplitude can be interpreted as the area by which each particle is reduced through openings with disks of increasing radii.

As expected, these pattern spectra are very different. The curve for particle *A* tends to take larger values for smaller radii, which reflects the more rugged appearance of this particle's boundary. On the other hand, the curve for particle *B* takes larger value for greater radii, which is consistent with the smooth appearance of the particle.

Based on these curves, we experimented with simple classification schemes. Without using classification techniques nearly as sophisticated as used in [14], we were able to extract correct Krumbein roundness from pattern spectra with an accuracy close to 100%.

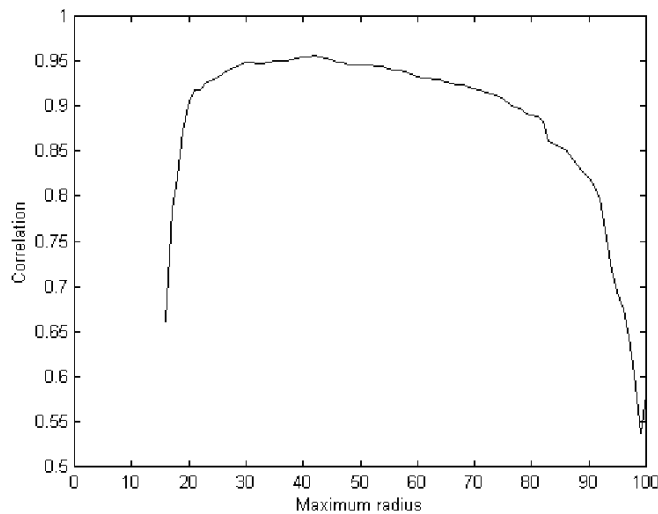


Fig. 8. Correlation between the area removed through opening with a disk and the Krumbein roundness, as a function of disk radius, normalized between 0 and 100.

Additionally, we were curious to know how far we could go using a *single* opening size. We did the following: for each radius N in our normalized scale $[1; 100]$, we computed the sum of the N first pattern spectra value. This is nothing more than computing, for each N , the proportion of the original particle that is removed by opening of radius $N\%$. We found that for $N = 42$, the correlation between this value and the Krumbein roundness value was equal to 0.96. Furthermore, any value of N between approximately 30 and 60 resulted in a correlation value greater or equal to 0.94. These results are shown in Fig. 8.

We can only conclude that, when it comes to roundness, all that the Krumbein chart really captures is the relative area of a particle that is removed through opening with a disk of radius approximately equal to 40% or 50% of the radius of the maximal inscribed disk.

Intuitively, the reason why openings with small disks do not correlate well with the Krumbein notion of roundness is because they mostly capture particle boundary texture, and are sometimes overly sensitive to digitization artifacts. On the other hand, when the radius of the disk approaches that of the maximal inscribed disk, then the method is too sensitive to sphericity. In fact, we hypothesize that the traditional notion of particle sphericity (see § 1) is highly correlated with the relative area that is removed through opening with the maximal inscribed disk. Furthermore, surface texture, which is the third property typically used to describe particle shapes [1], could itself be characterized using openings with smaller disks. We therefore believe that granulometric analysis can be used as a means to extract *all* the essential properties associated with rock particle shapes.

2.2. 1-D GRANULOMETRY ON POLAR COORDINATE CURVE

The roundness estimation method proposed in the previous section does not involve the somewhat expensive step of computing a granulometric curve based on openings with disks. Yet, it involves two nontrivial steps: (1) extraction of the radius of the maximal inscribed disk, which can be done using a Euclidean distance function algorithm (see, e.g., [2]); (2) opening with a fairly large disk.

We can speed this roundness estimation considerably by noticing that, for the relatively simple particle shapes of the Krumbein chart, opening a particle with a disk of radius r of the particle is roughly equivalent to *opening the polar coordinate representation of its outline, or ρ -curve (see Fig. 3) by a horizontal line segment*. Intuitively, all we are doing is working in the (ρ, θ) polar coordinate space, where a circle becomes a horizontal line segment. In general, one cannot compute a circular opening as a linear opening of the ρ -curve: for elongated shapes, nonconvex shapes, and any somewhat convoluted shape, this process produces results that bare little similarity to circular openings. However, as we prove below, this method does provide a new roundness estimation technique whose accuracy is on par with that of the method proposed in the previous section.

The speed gain comes from the fact that, using this approach, we turned a 2D problem into a much simpler 1D problem: instead of computing a complex morphological operation on a 2D image, we now only have to compute a simple linear opening on a 1D image with, in our case, 1024 θ samples (we use the same input data as used by the Diepenbroek method, see § 1.2). Note that in order to avoid undesirable edge effects, we have to account for the periodicity of the polar coordinates by replicating the signal on the left and on the right—though obviously, opening values are only measured over the $[0; 1023]$ range.

Our segment lengths ranged from 1 to 1024, which is the length of the ρ -curve R . As expected, and consistent with what was found using the 2D method described in the previous section, our experiments showed that a single opening size was sufficient to get excellent correlation with the Krumbein “groundtruth” roundness. Specifically, for each l , $1 \leq l \leq 1024$, we opened the ρ -curve R with a (horizontal) line segment of length l , thereby producing curve $\gamma_l(R)$. We then computed

$$G(l) = \frac{\sum_{\theta=0}^{1023} R(\theta) - \gamma_l(R(\theta))}{\sum_{\theta=0}^{1023} R(\theta)}$$

that is, the relative “volume” of the ρ -curve that is removed by opening of length l . We then normalized the curve $G(l)$ so the l values were in the range $[0; 100]$. This is consistent with what was done in the previous section.

As in § 2.1, we correlated for each l the values $G(l)$ obtained for all 81 particles with their Krumbein roundness value. We found that for values of l anywhere between 30 and 70 (in the normalized scale $[0; 100]$), there was an excellent correlation of 0.9 or more. This is on par with the findings of the previous section. Furthermore, as illustrated by Fig. 9, the correlation curve we obtain now (correlation as a function of l , for $1 \leq l \leq 100$) is almost identical to that previously obtained using circular openings.

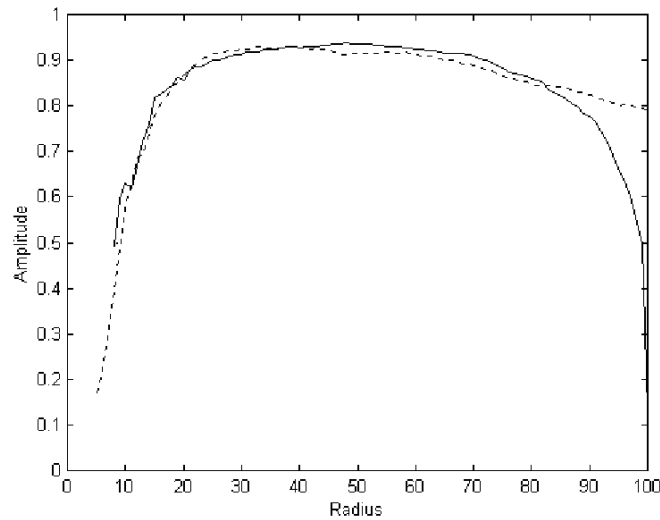


Fig. 9. Correlation curves for method based on 1D opening of polar coordinate profile (dotted line), and method based on circular opening of particle shape. The curves are almost identical and exhibit excellent correlation values in the [30;70] input range.

3. Results, Conclusion, Future Work

In practice, roundness is not only determined for individual particles but the mean roundness values of populations of particles are also determined. The Krumbein chart profiles were therefore divided into 9 populations with each population consisting of profiles that have the same roundness values. The mean roundness of each population was then determined using the different methods. The correlation between the mean roundness and actual roundness values are given in the following table in column **CoMR**. The correlation between the individually determined roundness and actual roundness values are given in column **Corr**.

Method	Corr	CoMR
Diepenbroek	0.94	0.967
Circular Opening	0.96	0.986
Linear Opening of ρ -curve	0.93	0.952

It is clear from this table that the results obtained with the two methods we proposed in this paper are either comparable or slightly superior to those obtained with the method of Diepenbroek *et al* [3]. Method #1 is slightly more accurate, while method #2 is significantly faster. Overall, we believe that both methods have the following advantages:

- Computational efficiency
- Accuracy

- Possibility of extracting particle roundness, but also sphericity and surface texture with the same method

The work reported here now needs to be followed with a more complete study. Analytical formulas for obtaining roundness from openings of specified relative size should be derived, for both methods. Accuracy could also be further improved by combining several opening sizes, that is, using more than just 1 granulometric value (e.g., 30%, 50%, and 70%). Furthermore, the study should be extended to encompass sphericity and surface texture. We envision that these granulometric methods will provide an efficient and accurate computational framework to study sedimentary rock particle shapes.

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